SHORT NOTES

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MATHEMATICAL MODEL OF A SUCCESSFUL STOCK MARKET GAME

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All available predictive models of stock market trade (like regression or statistical analysis, for instance) are based on studying of price fluctuation. This article proposes a new model of a successful stock market strategy based on studying of the behavior of the largest successful players. The main point of this model is that a relatively weak player repeats the actions of stronger players in the same fashion as in a race after leader a cyclist following a motorbike reaches greater velocity. We represent the leader as a vector in the nonnegative orthant $\mathbb{R}^n_+$ depending on the most successful traders (hedge funds). When buying and selling stocks, we should always keep the vector of own resources collinear to the leader's. This strategy will not yield significant profit, but it prevents considerable loss.

Keywords: stock market trade; hedge funds; race after leader.

Introduction. If market is the soul of every economy, then stock exchange is market's beloved child. Two relatively reliable methods for successful stock trade are available: regression analysis and statistical analysis. Both methods rest on studying of rate (price) fluctuation. We propose one more method for successful game at stock market, based on analysis of the behavior of the largest successful players. In this article we present only the foundations of this new mathematical model, which we call *race after leader*. The gist of this model is that to achieve better results in his sport a relatively weak participant (for instance, on a bicycle) follows a stronger one (for instance, on a motorcycle). The difference of our race after leader from its namesake in sports is that the leader does not even suspect that a follower exists, while in sports they must team up. In subsequent publications we will provide the mathematics behind the race after leader model together with examples to illustrate it.

In order to give considerable validity to the foundations of our new mathematical model, we start from basics. Recall first of all that the science of economics studies the processes of production and distribution (consumption) of material benefits and services, as well as the processes connecting them, i.e., finance. In accordance with the objects studied in modern economic theories, three paradigms are commonly distinguished, in each of which one of these types of processes is regarded as principal, while the other two as secondary and depending on the principal one. The paradigm emphasizing the processes of distribution was Historically the first. Adam Smith [1] is considered its father, and it is called the "classical political economy". Note that the name appeared a century later in Walras works [2]. By now this paradigm turned to "neoclassical" and even "post-neoclassical" political economy and formed a new branch of mathematics called mathematical economics [3]. Since our exposition will be in the framework of this paradigm, we now leave it to describe two other paradigms.

The next paradigm, based on studying of the processes of production, was developed by Karl Marx [4], and therefore is named as "Marxist political economy", or simply "Marxism". Owing its appearance to the mass production of the 19th century, this paradigm at first reflected the state of economy sufficiently adequately. However, serious doubts in its relevance arose since then. For instance, the growth of the profit rate postulated by Marxism (making the poor poorer and the rich richer) nowadays is not observed in the developed countries. Nevertheless, Leontieff's input-output model [5] created in the framework of this paradigm is still applicable [6, 7].

The third and last paradigm, based on studying of financial processes, was developed by John Maynard Keynes [8], and therefore is known as "Keynesian economics". Its main claim amounts to the requirement that the state regulates the economy by changing the bank loan rate. Meeting this requirement, expanding public works, and militarizing the economy, the USA managed to escape from the Great Depression.

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1. Market Analysis. To consider the market of capital issues (that is, stock market) from the viewpoint of classical political economy (more exactly, its mathematical representation), we should describe the goods and participants, as well as explain the phenomenon of instantly spreading information. Let us agree right away that for the sake of simplicity under goods we consider only the shares of companies, for a while leaving aside the markets of bonds, promissory notes, and other debt securities. (Here we diverge from the Keynesian economics.) Assume that the number of shares is measurable, it means that shares are not sold and bought by the piece. Furthermore, while trading shares, we make no distinctions in the format; that is, spots, forwards, futures, and so on are indistinguishable for us. (Since all transactions depend on share prices rather than their denomination, here we diverge from Marxism.) Thus, $n \in \mathbb{N}$ (virtual) facilities offer shares for sale at our (virtual) exchange. We express this situation in the nonnegative orthant

$$\mathbb{R}^n_+ = \{x \in \mathbb{R}^n : \; x_k \leq 0, \; k = 1, 2, \ldots, n\}$$

of the space $\mathbb{R}^n$.

The prototypes of our participants are real hedge funds, that is, companies buying and selling shares to make profits. Such companies exist; let us mention here a few classical examples. The first is the Graham Newman Investment Fund, founded by the father of investment business, Benjamin Graham (1894–1976) in 1926. Already in 1928 this company reached the average annual return of 25,7%. In the 1930s, having abandoned the most aggressive and high-risk transactions, Graham Newman survived the Great Depression with the losses of 50%. Subsequently the average annual profit grew to 17%. In 1956 B. Graham stepped down.

The second is one of the main hedge funds, Berkshire Hathaway of Warren Buffett (b. 1930), who was a student of Graham at Columbia University and then worked with him. In 2008 Buffett headed the list of the world's richest people according to the Forbes magazine. In 2010 Buffett announced the donation of 50% of his assets (about $37 billion) to five charities, which is the largest donation in history. A large part went to the fund of Bill and Melinda Gates.

Finally, the third is the Quantum Fund NV, the largest fund in the Quantum Group of Funds controlled by George Soros (b. 1930). Soros named it in honor of Werner Heisenberg, the author of the uncertainty principle of quantum mechanics. Soros's fortune in September of 2014 was about $28,7 billion. He spent more than $5 billion on charity, of which more than a billion in Russia.

Therefore, associate to a participant with index $m \in \mathbb{N}$ (that is, some hegde raid) a vector $x^m \in \mathbb{R}^n_+$, each component x^m_k of which means the number of shares of k-th company for $k = 1, 2, \ldots, n$, owned by the corresponding participant, for instance, at time t_0. At time $t_1 > t_0$, after some shares are bought and others are sold, the vector x^m changes. We can learn about the components of x^m and their change, for instance, in the USA thanks to the Dodd–Frank Act. This lengthy (more than 2300 pages) legislation regulates the operation of hedge funds in detail; in particular, it requires monthly reports on their financial activity. Thus, the Dodd–Frank Act makes fast if not instantaneous circulation of data possible.

2. Race after Leader. Now we are ready to construct our mathematical model. Describe our algorithm in steps.

— Enumerate all hedge funds arbitrarily. Associate to the hedge fund with index $m \in \mathbb{N}$ its assets A_m a year ago, the profit P_m made in the previous year by buying and selling shares, and the *success coefficient* $æ_m = P_m/A_m$.

— Choose hedge funds with positive success coefficient. The sample S is of arbitrary size, it can contain a unique hedge fund with maximal $æ_m$, or all hedge funds with positive $æ_m$. Enumerate the hedge funds in S in the order of increasing of $æ_m$.

— Associate to each hedge fund in S a vector $x^m \in \mathbb{R}^n_+$ constructed using the recipes of the previous section. (If we do this in the USA then it is better to construct x^m for $m \in S$ in the beginning of the month.) Construct the vector

$$L = \sum_{m \in S} æ_m x^m,$$

which we call *the leader*.

— Create our own hedge fund with the asset vector $\ell \in \mathbb{R}^n_+$ collinear to L. The coefficient of collinearity is obviously positive and less than 1.

— Now we should sell and buy shares keeping ℓ collinear to L. It is clear that, acting in this way, we make less profit than the most successful trader, but greater than the least successful one. In addition, the algorithm is capable of preventing bankruptcy even in the case that someone in the sample becomes involved in high-risk operations and goes bankrupt (as in [9] for instance), provided, however, that S contains more than a single participant.

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МАТЕМАТИЧЕСКАЯ МОДЕЛЬ УСПЕШНОЙ ИГРЫ НА ФОНДОВОЙ БИРЖЕ

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Все известные прогностические модели биржевых спекуляций (такие как например, регрессионный или статистический анализ) основаны на изучении колебаний курсов ценных бумаг. В статье предложена новая модель успешной игры на фондовой бирже, основанная на изучении поведения крупнейших удачливых игроков. Суть предлагаемой модели заключается в том, что относительно слабый игрок повторяет действия сильного игрока подобно тому, как в спортивной «гонке за лидером» велосипедист, укрываясь за мотоциклистом, развивает бо́льшую скорость. Мы под «лидером» понимаем вектор в неотрицательном ортанте $\mathbb{R}^n_+$, который строится в зависимости от наиболее удачливых биржевых спекулянтов (хедж-фондов). Вектор собственных ресурсов путем купли-продажи ценных бумаг всегда следует держать коллинеарным «лидеру». Такая стратегия не приведет к значительному выигрышу, но и не позволит случиться значительному проигрышу.

Ключевые слова: биржевые спекуляции; хедж-фонды; гонка за лидером.

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