THE PROBLEM OF OPTIMAL CONTROL OVER SOLUTIONS OF THE NONSTATIONARY BARENBLATT – ZHELTOV – COCHINA MODEL

***M.A. Sagadeeva**, South Ural State University, Chelyabinsk, Russian Federation, sagadeeva_ma@mail.ru,*
***A.D. Badoyan**, South Ural State University, Chelyabinsk, Russian Federation, badoyanani@mail.ru*

The problem of optimal control over solutions for the Barenblatt – Zheltov – Cochina nonstationary equation with Showalter – Sidorov condition is studied in this article. This study presents a numerical algorithm for solving optimal control problems. In the final part there is a numerical experiment for Barenblatt – Zheltov – Cochin non-stationary equation considered on a rectangle.

Keywords: non-stationary Sobolev equation, the optimal control problem, Showalter – Sidorov condition, Barenblatt – Zheltov – Cochina model.

Introduction

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with a bound $\partial\Omega$ from class C^∞. Consider the Dirichlet problem in the cylinder $\Omega \times \mathbb{R}$ for Sobolev type equation

$$(\lambda - \Delta)x_t = a(t)\Delta x + u, \tag{1}$$

that simulates the dynamics of the fluid pressure filters in fractured porous medium [1]. In equation $\lambda \in \mathbb{R}$ is a real parameter and a scalar function $a: \overline{\mathbb{R}}_+ \to \mathbb{R}_+$, characterize the environment, and λ can take negative values. Vector-function $u: \mathbb{R} \to L_2(\Omega)$ is a control function and characterizes the out influences on the system.

Equation (1) belongs to a class of Sobolev type equations [2], constitutes a large class of non-classical equations of mathematical physics [3–5]. Let's note that in contrast to earlier equation (1) studies (see for example [2]), we consider the equation (1) with a coefficient that depends on time.

Introduce the quality functional

$$J(u) = \sum_{q=0}^{1}\left(\int_0^T \left\| z^{(q)}(t) - z_d^{(q)}(t)\right\|_{\mathfrak{Z}}^2 dt + \int_0^T \langle u^{(q)}(t), u^{(q)}(t)\rangle_{\mathfrak{U}} dt\right), z = Cx, \tag{2}$$

where $\mathfrak{U}$ and $\mathfrak{Z}$ are Hilbert spaces, $T \in \mathbb{R}_+$, $C \in \mathcal{C\ell}(\mathfrak{X}, \mathfrak{Z})$, z_d is planned state of the system. Our task is to find the optimal control v, which minimizes functional (2) at a closed convex subset for equation (1) with Showalter – Sidorov condition [6]:

$$P(x(0) - x_0) = 0. \tag{3}$$

1. Abstract results

Let $\mathfrak{X}$, $\mathfrak{Y}$ be Banach spaces, operator $L \in \mathcal{L}(\mathfrak{X}; \mathfrak{Y})$ with nontrivial kernel $\ker L \neq \{0\}$, operator $M \in \mathcal{C\ell}(\mathfrak{X}; \mathfrak{Y})$.

The sets $\rho^L(M) = \{\mu \in \mathbb{C}: (\mu L - M)^{-1} \in \mathcal{L}(\mathfrak{X}, \mathfrak{Y})\}$ and $\sigma^L(M) = \mathbb{C}\backslash\rho^L(M)$ are called respectively *L-resolvent set* and *L-spectrum of the operator M*.

On condition $\ker L \cap \ker M = \{0\}$, then $\rho^L(M) = \emptyset$.

Operator-function $(\mu L - M)^{-1}$, $R_\mu^L(M) = (\mu L - M)^{-1}L$, $L_\mu^L(M) = L(\mu L - M)^{-1}$ are called respectively a *resolvent*, *right resolvent*, and *left resolvent* of an operator M *with respect to* the operator L (or briefly *L-resolvent*, *right L-resolvent*, and *left L-resolvent* of an operator M correspondingly).

Definition 1. Operator M is called *spectrally bounded with respect to* the operator L (or briefly *(L, σ)-bounded*), if $\exists r > 0\ \forall \mu \in \mathbb{C}\ (|\mu| > r) \Longrightarrow (\mu \in \rho^L(M))$.

Let the operator M be (L, σ)-bounded, choose in complex subspace $\mathbb{C}$ the counter $\gamma = \{\mu \in \mathbb{C}: |\mu| = R > r\}$. Let us consider integrals of F. Riss type

$$P = \frac{1}{2\pi i}\int_\gamma (\mu L - M)^{-1}L d\mu, Q = \frac{1}{2\pi i}\int_\gamma L(\mu L - M)^{-1} d\mu.$$

Operators P and Q are projectors. Let's denote $\mathfrak{X}^0 = \ker P$, $\mathfrak{Y}^0 = \ker Q$, $\mathfrak{X}^1 = \operatorname{im} P$, $\mathfrak{Y}^1 = \operatorname{im} Q$, then $\mathfrak{X} = \mathfrak{X}^0 \oplus \mathfrak{X}^1$, $\mathfrak{Y} = \mathfrak{Y}^0 \oplus \mathfrak{Y}^1$. Let the restriction of the operator $L(M)$ to $\mathfrak{X}^k$ $\left(\operatorname{dom} M_k = \operatorname{dom} M \cap \mathfrak{X}^k\right)$, $k = 0, 1$ is denoted by $L_k(M_k)$.

In addition through $\sigma_k^L(M_k)$ is denoted the set $\mathbb{C} \backslash \rho^{L_k}(M_k)$ – L_k-spectrum of the operator M_k.

Theorem 1. [2] *Let operator M be (L,σ)-bounded. Then*

1) $L_k \in \mathcal{L}(\mathfrak{X}^k; \mathfrak{Y}^k), k =0, 1$;

2) $M_0 \in \mathcal{C}\ell(\mathfrak{X}^0; \mathfrak{Y}^0),\ M_1 \in \mathcal{C}\ell(\mathfrak{X}^1; \mathfrak{Y}^1)$;

3) *there exists an operator* $L_1^{-1} \in \mathcal{L}(\mathfrak{Y}^1; \mathfrak{X}^1)$;

4) $\sigma_0^L(M_k) = \emptyset$, *in particular, there exists an operator* $M_0^{-1} \in \mathcal{L}(\mathfrak{Y}^0; \mathfrak{X}^0)$;

5) *there is an analytic solving semigroup* $\{X^t \in \mathcal{L}(\mathfrak{X}): t \in \mathbb{R}\}$ *of equation* $L\dot{x}(t) = Mx(t)$ ($\{Y^t \in \mathcal{L}(\mathfrak{Y}): t \in \mathbb{R}\}$ for $L(\alpha L - M)^{-1}\dot{y}(t) = M(\alpha L - M)^{-1}y(t)$ *when* $\alpha \in \rho^L(M)$), form

$$X^t = e^{tL_1^{-1}M_1}P = \frac{1}{2\pi i}\int_\gamma R_\mu^L(M)e^{\mu t}d\mu \left(Y^t = e^{tM_1L_1^{-1}}Q = \frac{1}{2\pi i}\int_\gamma L_\mu^L(M)e^{\mu t}d\mu\right).$$

Definition 2. If operator M is (L,σ)-bounded let introduce operators $H = M_0^{-1}L_0 \in \mathcal{L}(\mathfrak{X}^0)$ and $S = L_1^{-1}M_1 \in \mathcal{L}(\mathfrak{X}^1)$. In this case

• if $H = \mathbb{O}$, then the point ∞ is called a *removable singularity* of the L-resolvent of the operator M and operator M is $(L, 0)$-*bounded*;

• if $H^p \neq \mathbb{O}$, a $H^{p+1} = \mathbb{O}$, then the point ∞ is called pole of order $p \in \mathbb{N}$ of the L-resolvent of the operator M and operator M is (L,p)-*bounded*;

• if $\forall p \in \mathbb{N}\ H^p \neq \mathbb{O}$, then the point ∞ is called *essential singularity* of the L-resolvent of the operator M and operator M is (L,∞)-*bounded.*

Through $\mathbb{O}$ is denoted the zero operator defined on the space $\mathfrak{X}^0$.

Consider the space $H^{p+1}(\mathfrak{Y}) = \{\xi \in L_2(\Omega; \mathfrak{Y}): \xi^{(p+1)} \in L_2(\Omega; \mathfrak{Y}),\ p \in (\mathbb{N} \cup 0), \Omega \in \mathbb{R}^n\}$, that is Hilbert space because of $\mathfrak{Y}$ is Hilbert space with the scalar product

$$[\xi, \eta] = \sum_{q=0}^{p+1} \int_\Omega \langle \xi^{(q)}, \eta^{(q)} \rangle_{\mathfrak{Y}}\, dt.$$

Let $\mathfrak{X}, \mathfrak{Y}, \mathfrak{U}$ be Hilbert space. In the domain $\Omega \in \mathbb{R}^n$ consider Showalter–Sidorov (4) condition for the Sobolev type equation [2]

$$\dot{x}(t) = a(t)Mx(t) + u(t). \tag{4}$$

Here $L \in \mathcal{L}(\mathfrak{X}; \mathfrak{Y})$, $M \in \mathcal{C}\ell(\mathfrak{X}; \mathfrak{Y})$, scalar function $a: \overline{\mathbb{R}}_+ \to \mathbb{R}_+$, function $u: \mathbb{R} \to L_2(\Omega)$ is a control function.

Theorem 2. *Let the operator be M (L,p)-bounded*, $p \in \{0\} \cup \mathbb{N}$, *then for every* $x_0 \in \mathfrak{X}$, $a \in C^{p+1}([0,T); \mathbb{R}_+)$ ($[0,T) \subset \mathbb{R}_+$, $T < +\infty$) *there exists a unique solution* $x \in H^1(\mathfrak{X})$ *of the problem* (3), (4) *of the form*

$$x(t) = X^{\int_0^t a(\xi)d\xi}Px_0 + \int_0^t X^{\int_s^t a(\xi)d\xi}L_1^{-1}Qu(s)ds + \sum_{k=0}^p H^k L_1^{-1}(I - Q)\left(\frac{1}{a(t)}\frac{d}{dt}\right)^k \frac{u(t)}{a(t)}, \tag{5}$$

where the expression $\left(\frac{1}{a(t)}\frac{d}{dt}\right)^k$ *in the last component is consistent application of the operator k times.*

The proof of this theorem in a more general case is given in [7].

Let us consider the optimal control problem. Separate in space $H^{p+1}(\mathfrak{U})$ closed and convex subset $H_\partial^{p+1}(\mathfrak{U}) = \mathfrak{U}_\partial$ is the set of admissible controls.

Definition 3. Vector function v is *optimal control* of the solutions of problem (3), (4) with functional (2), if

$$J(v) = \min_{u \in \mathfrak{U}_\partial} J(u), \tag{6}$$

where $x \in H^1(\mathfrak{X})$, constructed from $u \in \mathfrak{U}_\partial$, is the solution of problem (1), (3).

Theorem 3. *Let operator M* be (L,p)-*bounded*, $p \in \{0\} \cup \mathbb{N}$, *function* $a \in C^{p+1}(\overline{\mathbb{R}}_+; \mathbb{R}_+)$ *is separated from zero. Then for every* $x_0 \in \mathfrak{X}$ *there exists unique optimal control* $v \in \mathfrak{U}_\partial$ *for problems* (3), (4), (6) *with functional* (2).

Theorem 3 follows from the form of the solution (5). For details, see [7].

The first results of the optimal control for linear Sobolev type equation can be found in [2]. The optimal control problem for non-linear Sobolev type equation is considered in the monograph [8]. Also recently the optimal control over solutions of the Sobolev type equations considered for various models [9, 10]. As for the non-stationary Sobolev type equations for equation (4) in the case of relatively p-sectorial [2] is considered in work [7], and in a more general setting, where the operator M is an operator-valued function of the variable t, the optimal control problem is considered in [11].

Following [12] let's describe the approximate solution of the problem of the optimal measurement. Replace the control space for finite-dimensional space $\mathfrak{U}^l = H_l^{p+1}(\mathbb{R}^n)$ vector-polynomials of the form $u^l = u^l(t)$, where

$$u^l = \mathrm{col}\left(\sum_{j=0}^{l} c_{1j}t^j, \sum_{j=0}^{l} c_{2j}t^j, \dots, \sum_{j=0}^{l} c_{nj}t^j, \dots\right).$$

Counting the form (5), it is necessary that $l > p$. Substituting u^l instead u in (2), (5) and considering the optimal control problem $J(v^l) = \min_{u^l \in \mathfrak{U}_\partial} J(u^l)$ let's obtain the solution (v^l, x^l), where $x^l = x(v^l, t)$.

2. Barenblatt – Zheltov – Kochin model

Let consider Barenvlatt – Zheltov – Kochin equation [1]

$$(\lambda - \Delta)x_t = a\Delta x + u, \tag{7}$$

which simulates the dynamics of the fluid pressure filters in fractured porous medium Furthermore, equation (7) describes flow of the second-order fluid [13], process of moisture transfer in the soil [15] and other.

In equation $\lambda \in \mathbb{R}$ is a real parameter characterizes the environment; λ can take negative values [2]. In the Barenvlatt – Zheltov – Kochin model (7) parameter a is composite parameter value [1], depending on the fluid properties and fluid permeability of the system and corresponds to cracks and porosity and compressibility of blocks. Its value is determined by the formula

$$a = \frac{\alpha_1}{\mu(\beta_1 + m_0\beta)},$$

where α_1 is the dimensionless characteristic fractured medium, μ is the liquid viscosity, m_0 is porosity value blocks at standard pressure, β_1 is compressibility coefficient of blocks, β is compressibility coefficient of liquid [1]. To improve the adequacy of the model to real physical processes, coefficient β_1 and β advisable to take time-dependent and consider this parameter a as a time dependent scalar function $a: \overline{\mathbb{R}}_+ \to \mathbb{R}_+$.

For the reduction of (1) to the Sobolev type equation (4) let's take a bounded domain $\Omega \in \mathbb{R}^n$ with boundary $\partial\Omega$ of class C^∞. Let find the function $x = x(t, s_1, s_2)$, defined in the cylinder $\Omega \times \mathbb{R}$ satisfying the equation (1), the initial condition (3) and the boundary condition

$$x(t,s) = 0, \ s \in \partial\Omega. \tag{8}$$

In this case problems (3), (8) for equation (1) are reduced to the abstract problem (3) for the equation (4), taking as Sobolev spaces $\mathfrak{X}$, $\mathfrak{Y}$, where

$$\mathfrak{X} = \{x \in W_k^{p+2}(\Omega): x(s) = 0, s \in \partial\Omega\}, \mathfrak{Y} = W_k^p(\Omega), 1 < p < \infty, k = 0, 1, \dots \tag{9}$$

Then the operators take the form

$$L = \lambda - \Delta: \mathfrak{X} \to \mathfrak{Y}, M = \Delta: \mathfrak{X} \to \mathfrak{Y}. \tag{10}$$

Lemma 1. [2] *Let the spaces* $\mathfrak{X}$, $\mathfrak{Y}$ *is defined in* (9), the operators L, M *is defined in* (10). *Then operator M is* $(L, 0)$*-bounded.*

In condition lemma 1 the existence theorem of solution for the Barenblatt – Zheltov – Cochin equation is fair.

Theorem 4. *Operator M is* $(L, 0)$*-bounded,* $\lambda \in \mathbb{R}$, $a \in C^1(\overline{\mathbb{R}}_+; \mathbb{R}_+)$, $u \in H^1(\mathfrak{U})$. *Then there exists a unique solution the problems* (3), (8) *for equation* (1) *represented by the form*

$$x(t) = \sum_{k\in\mathbb{N}:\lambda_k \neq \lambda}^{\infty} e^{\frac{\lambda_k}{\lambda-\lambda_k}\int_o^t a(\tau)d\tau}(x_0, \varphi_k)\varphi_k + \sum_{k\in\mathbb{N}:\lambda_k \neq \lambda}^{\infty} \int_o^t e^{\frac{\lambda_k}{\lambda-\lambda_k}\int_s^t a(\tau)d\tau} \frac{(u(s), \varphi_k)}{\lambda - \lambda_k}\varphi_k ds -$$

$$- \frac{1}{a(t)\lambda}\sum_{k\in\mathbb{N}:\lambda_k = \lambda}^{\infty}(u, \varphi_k)\varphi_k. \tag{11}$$

Here $\{\varphi_k\}$ and $\{\lambda_k\}$ are the set of orthon set of orthonormal eigenfunctions and the corresponding eigenvalues of the Dirichlet problem for the Laplace operator in Ω, indexed descending eigenvalues with multiplicities. Here accounting the possibility of getting the parameter λ in the relative L-spectrum of the operator M, where $\lambda = \lambda_m$.

3. Numerical experiment

Based on the obtained results numerical method for solving optimal control problem has been designed for non-stationary model Barenblatt-Zheltov-Kochina in some domain.

Let consider the basic steps of an algorithm for finding the optimal control problem solutions.

Step 1. Input parameters λ, $a(t)$, the boundary condition $x(t,s)=0,\ s\in\partial\Omega$, initial condition of problem x_0 and planned state system x_d.

Step 2. Generation type component of optimal control in the form of a polynomial

$u^l(t)=\mathrm{col}\left(\sum_{j=0}^{l}c_{1j}t^j,\sum_{j=0}^{l}c_{2j}t^j,\dots,\sum_{j=0}^{l}c_{nj}t^j,\dots\right)$.

Step 3. Computation of the solution of the problem Showalter-Sidorova (3) for the equation (1) with the condition (8) in the form

$$x_N(t)=\sum_{k\in\mathbb{N}:\lambda_k\neq\lambda}^{N}e^{\frac{\lambda_k}{\lambda-\lambda_k}\int_o^t a(\tau)d\tau}(x_0,\varphi_k)\varphi_k+\sum_{k\in\mathbb{N}:\lambda_k\neq\lambda}^{N}\int_o^t e^{\frac{\lambda_k}{\lambda-\lambda_k}\int_s^t a(\tau)d\tau}\frac{(u^l(s),\varphi_k)}{\lambda-\lambda_k}\varphi_k ds-$$

$$-\frac{1}{a(t)\lambda}\sum_{k\in\mathbb{N}:\lambda_k=\lambda}^{\infty}(u^l,\varphi_m)\varphi_m.$$

Step 4. Building the functional

$$J(u^l)=\sum_{q=0}^{1}\left(\int_0^T\left\|x^{(q)}(t)-x_d^{(q)}(t)\right\|_3^2dt+\int_0^T\langle(u^l(t))^{(q)},(u^l(t))^{(q)}\rangle_{\mathfrak{U}}\,dt\right)$$

and closed convex subset of admissible controls $\left\|u^l(t)\right\|_3^2<1$.

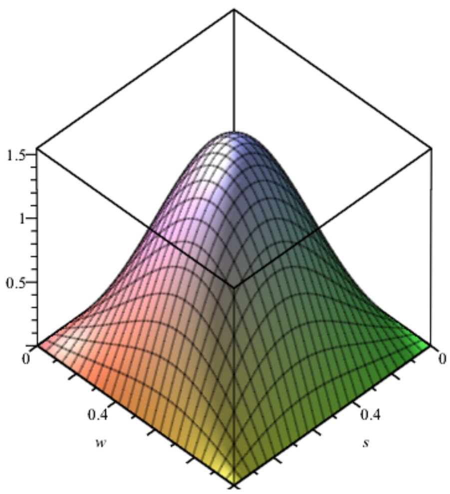
Step 5. On the subset of admissible controls with built-in procedure for finding extrema of functions of several variables in a system of 14 Maple calculated minimum of the functional $J(u^l)$.

Let consider an example illustrating the results obtained above. Required to find the solution (1), (3), (8) for the following parameters. Let $l=2$, $N=4$. Domain $\Omega=\{(s_1,s_2)\in\mathbb{R}^2:0\leq s_1\leq 1,0\leq\leq s_2\leq 1\}\subset\mathbb{R}^2$. The initial condition is given in the form

$x(0,s_1,s_2)=\sin(\pi s_1)\sin(\pi s_2)+\sin(2\pi s_1)\sin(\pi s_2)+$
$+\sin(\pi s_1)\sin(2\pi s_2)+\sin(2\pi s_1)\sin(2\pi s_2)$.

Planned state at the final time has the form (fig. 1)

$x_d(t,s_1,s_2)=(t+1)(\sin(\pi s_1)\sin(\pi s_2)+\sin(2\pi s_1)\sin(\pi s_2)+$
$+\sin(\pi s_1)\sin(2\pi s_2)+\sin(2\pi s_1)\sin(2\pi s_2))$.



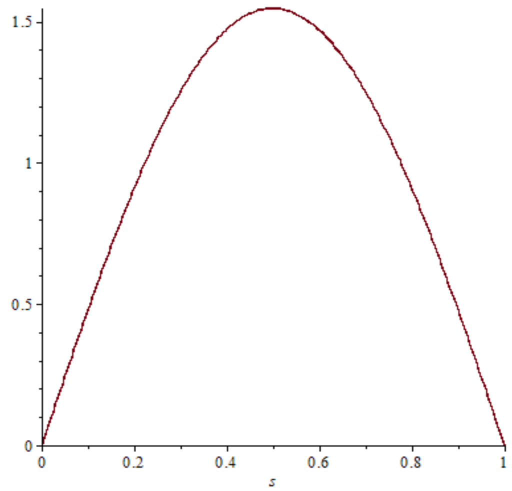


Fig. 1. Required state at the final time

The function $a(\tau)=\frac{1}{\tau+1}$, parameter $\lambda=-5\pi^2$ (coincides with a second eigenvalue), $t=1$. Substituting these parameters in (11) and solving the optimal control problem (6) with the functional (2) let find the function v.

The resulting solution of the problem of optimal control in the final time is shown in fig. 2.

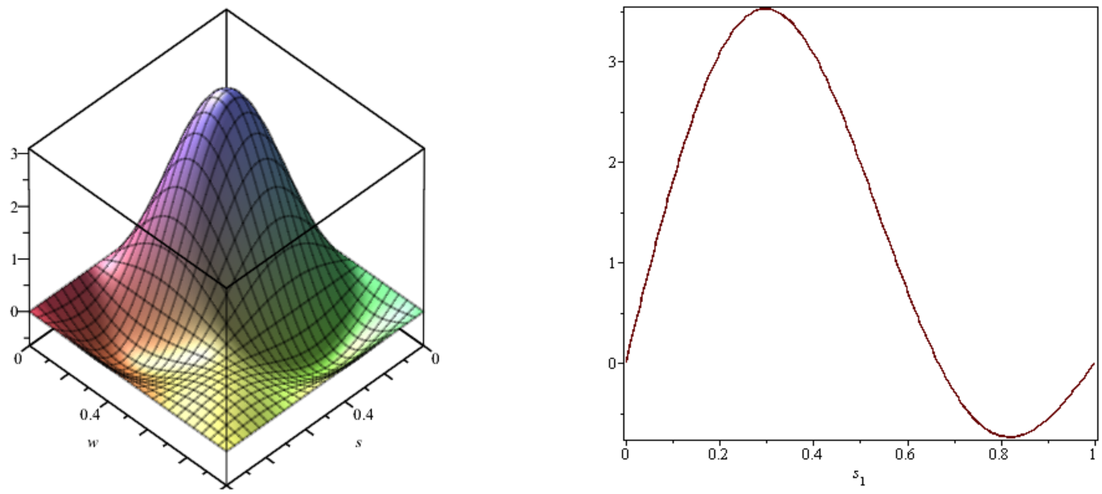


Fig. 2. Solution of the optimal control problem at the final time

Fig. 3 shows graphs of solutions of optimal control (solid line) and the planned state (dashed line). As seen from the results obtained by routine monitoring and close decision in the integral sense.

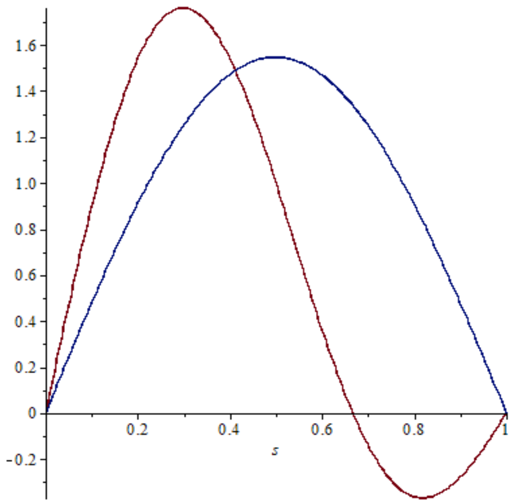


Fig. 3. Required observation and solution
of the optimal control problem at the final time

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ЗАДАЧА ОПТИМАЛЬНОГО УПРАВЛЕНИЯ РЕШЕНИЯМИ НЕСТАЦИОНАРНОЙ МОДЕЛИ БАРЕНБЛАТТА – ЖЕЛТОВА – КОЧИНОЙ

М.А. Сагадеева, А.Д. Бадоян

В статье рассматривается задача оптимального управления решениями задачи Шоуолтера – Сидорова для нестационарного уравнения Баренблатта – Желтова – Кочиной. В работе представлен алгоритм численного решения задачи оптимального управления. В заключительной части приводится вычисленный эксперимент для нестационарного уравнения Баренблатта – Желтова – Кочиной, рассмотренной на прямоугольнике.

Ключевые слова: нестационарные уравнения соболевского типа, задача оптимального управления, задача Шоуолтера – Сидорова, модель Баренблатта – Желтова – Кочиной.

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Сагадеева Минзиля Алмасовна, канд. физ.-мат. наук, доцент кафедры информационно-измерительной техники, Южно-Уральский государственный университет (г. Челябинск); sagadeeva_ma@mail.ru.

Бадоян Ани Давидовна, магистрант кафедры уравнений математической физики, Южно-Уральский государственный университет (г. Челябинск); badoyanani@mail.ru.

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